Time variation of the gravitational coupling constant in decrumpling cosmology

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Abstract

Within the framework of a model universe with time variable space dimension (TVSD) model, known as decrumpling or TVSD model, we study the time variation of the gravitational coupling constant. Using observational bounds on the present time variation of the gravitational Newton's constant in three-dimensional space we are able to obtain a constraint on the time variation of the gravitational coupling constant. As a result, the absolute value of the time variation of the gravitational coupling constant must be less than $\sim 10^{-11}\mathrm{yr}^{-1}$.

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1 Introduction

Usually in cosmological models based on higher dimensions the problem of dimensionality of the gravitational coupling constant is not tackled on, being tacitly assumed to be

$$\kappa = 8\pi G, \tag{1}$$

where G is the Newton's gravitational constant. This is of course a relationship being derived in $(3+1)$-dimensional spacetime. In this paper, we first review time variation of the gravitational coupling constant in all dimensions and then study time variation of the gravitational coupling constant in a model universe with time variable space dimensions (TVSD), known as decrumpling or TVSD model.

The plan of this paper is as follows. In Section 2, we review the gravitational coupling constant in all dimensions. In Section 3, we first review

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decrumpling or TVSD model and then obtain the time variation of the gravitational coupling constant in this model. Finally, we discuss our results and conclude in Section 4. We will use a natural unit system that sets k_B, c, and $\hbar$ are equal to one, so that $\ell_P = M_P^{-1} = \sqrt{G}$.

2 Gravitational coupling constant in all dimensions

Let us here review the gravitaional coupling constant in all dimensions (see Ref. [1]). Take the metric in $(D+1)$-dimensional spacetime in the following form

$$ds^2 = -dt^2 + a^2(t)d\Sigma_k^2, \tag{2}$$

where $d\Sigma_k^2$ is the line element for a D-manifold of constant curvature $k = -1, 0, +1$, corresponding to the closed, flat and hyperbolic spacelike sections, respectively. Using Eq.(2), we obtain[2]

$$R_{00} = (D-2)\nabla_D^2\phi, \tag{3}$$

where ∇_D is the ∇ operator in D-dimensional spaces. In $(3+1)$-dimensional spacetime, the Poisson equation is given by $\nabla^2\phi = 4\pi G\rho$. Applying Gauss law for a D-dimensional volume, we find the Poisson equation for arbitrary fixed dimension

$$\nabla_D^2\phi = S^{[D]}G_{(D+1)}\rho, \tag{4}$$

where $G_{(D+1)}$ is the $(D+1)$-dimensional Newton's constant and $S^{[D]}$ is the surface area of a unit sphere in D-dimensional spaces

$$S^{[D]} = \frac{2\pi^{D/2}}{\Gamma\left(\frac{D}{2}\right)}. \tag{5}$$

On the other hand from Eq.(2) we get

$$R_{00} = \left(\frac{D-2}{D-1}\right)\kappa_{(D+1)}\rho. \tag{6}$$

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[2]It is worth mentioning that in D-dimensional spaces the authors of Ref.[2] have considered $R_{00} = \nabla^2\phi$. For this reason, the results presented in Ref.[2] are not in agreement with the results presented here and Ref.[1].

Using Eqs.(3-6), we are led to the gravitational coupling constant in $(D+1)$-dimensional spacetime[3]

$$\kappa_{(D+1)} = (D-1)S^{[D]}G_{(D+1)} = \frac{2(D-1)\pi^{D/2}G_{(D+1)}}{\Gamma\left(\frac{D}{2}\right)}. \qquad (7)$$

Let us now obtain a relationship between the Newton's constant in $(D+1)$-dimensional spacetime and in the $(3+1)$-dimensional spacetime. Using the force laws in $(D+1)$- and $(3+1)$- dimensional spacetime, which are defined by

$$F_{(D+1)}(r) = G_{(D+1)}\frac{m_1 m_2}{r^{D-1}}, \qquad (8)$$

$$F_{(3+1)}(r) = G_{(3+1)}\frac{m_1 m_2}{r^2}, \qquad (9)$$

and the $(D+1)$-dimensional Gauss law one can derive (see Ref.[3])

$$G_{(3+1)} = \frac{S^{[D]}}{4\pi}\frac{G_{(D+1)}}{V^{[D-3]}}, \qquad (10)$$

where $V^{[D-3]}$ is the volume of $(D-3)$ extra spatial dimensions. Now, using Eqs.(7) and (10) we are led to

$$\kappa_{(D+1)} = 4\pi(D-1)G_{(3+1)}V^{[D-3]}. \qquad (11)$$

Our approach here to obtain the gravitational coupling constant in all dimensions is a model-independent approach and may be used for cosmological models in higher dimensions.

3 Time variation of gravitational coupling constant in the model

Let us briefly review decrumpling or TVSD model (for more details see Refs.[4]-[11]). Assume the universe consists of a fixed number $\bar{N}$ of universal cells having a characteristic length δ in each of their dimensions. The

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[3]For example, in the cases $(3+1)$, $(4+1)$ and $(5+1)$-dimensional spacetime we have $\kappa_{(3+1)} = 8\pi G_{(3+1)}$ (i.e. $\kappa = 8\pi G$), $\kappa_{(4+1)} = 6\pi^2 G_{(4+1)}$ and $\kappa_{(5+1)} = \frac{32\pi^2}{3}G_{(5+1)}$, respectively.

volume of the universe at the time t depends on the configuration of the cells. It is easily seen that [10]

$$\text{vol}_{D_t}(\text{cell}) = \text{vol}_{D_0}(\text{cell})\delta^{D_t-D_0}, \tag{12}$$

where the t subscript in D_t means D is to be as a function of time. Interpreting the radius of the universe, a, as the radius of gyration of a crumpled "universal surface", the volume of space can be written [10]

$$
\begin{aligned}
a^{D_t} &= \bar{N}\text{vol}_{D_t}(\text{cell}) \\
&= \bar{N}\text{vol}_{D_0}(\text{cell})\delta^{D_t-D_0} \\
&= a_0{}^{D_0}\delta^{D_t-D_0}
\end{aligned}
\tag{13}
$$

or

$$\left(\frac{a}{\delta}\right)^{D_t} = \left(\frac{a_0}{\delta}\right)^{D_0} = e^C, \tag{14}$$

where C is a universal positive constant. Its value has a strong influence on the dynamics of spacetime, for example on the dimension of space, say, at the Planck time. Hence, it has physical and cosmological consequences and may be determined by observations. The zero subscript in any quantity, e.g. in a_0 and D_0, denotes its present values. We coin the above relation as a "dimensional constraint" which relates the "scale factor" of the model universe to the space dimension. In our formulation, we consider the comoving length of the Hubble radius at present time to be equal to one. So the interpretation of the scale factor as a physical length is valid. The dimensional constraint can be written in this form

$$\frac{1}{D_t} = \frac{1}{C}\ln\left(\frac{a}{a_0}\right) + \frac{1}{D_0}. \tag{15}$$

It is seen that by expansion of the universe, the space dimension decreases. Time derivative of Eqs.(14) or (15) leads to

$$\dot{D}_t = -\frac{D_t^2\dot{a}}{Ca}. \tag{16}$$

It can be easily shown that the case of constant space dimension corresponds to when C tends to infinity. In other words, C depends on the number of fundamental cells. For $C \to +\infty$, the number of cells tends to infinity and

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Table 1: Values of C and δ for some values of D_P [4]-[10]. Time variation of space dimension today has also been calculated in terms of yr^{-1}.

D_P	C	δ (cm)	$\dot{D}_t\|_0$ (yr^{-1})
3	$+\infty$	0	0
4	1678.797	8.6158×10^{-216}	$-5.4827 \times 10^{-13} h_0$
10	599.571	1.4771×10^{-59}	$-1.5352 \times 10^{-12} h_0$
25	476.931	8.3810×10^{-42}	$-1.9299 \times 10^{-12} h_0$
$+\infty$	419.699	ℓ_P	$-2.1931 \times 10^{-12} h_0$

$\delta \to 0$. In this limit, the dependence between the space dimensions and the radius of the universe is removed, and consequently we have a constant space dimension.

We define D_P as the space dimension of the universe when the scale factor is equal to the Planck length ℓ_P. Taking $D_0 = 3$ and the scale of the universe today to be the present value of the Hubble radius H_0^{-1} and the space dimension at the Planck length to be $4, 10$, or 25, from Kaluza-Klein and superstring theories, we can obtain from Eqs. (15) and (16) the corresponding value of C and δ

$$\frac{1}{D_P} = \frac{1}{C} \ln\left(\frac{\ell_P}{a_0}\right) + \frac{1}{D_0} = \frac{1}{C} \ln\left(\frac{\ell_P}{H_0^{-1}}\right) + \frac{1}{3}, \tag{17}$$

$$\delta = a_0 e^{-C/D_0} = H_0^{-1} e^{-C/3}. \tag{18}$$

In Table 1, values of C, δ and also $\dot{D}_t\|_0$ for some interesting values of D_P are given. These values are calculated by assuming $D_0 = 3$ and $H_0^{-1} = 3000 h_0^{-1} \mathrm{Mpc} = 9.2503 \times 10^{27} h_0^{-1} \mathrm{cm}$, where $h_0 = 0.68 \pm 0.15$. Since the value of C and δ are not very sensitive to h_0 we take $h_0 = 1$.

Let us define the action of the model for the special Friedmann-Robertson-Walker (FRW) metric in an arbitrary fixed space dimension D, and then try to generalize it to variable dimension D_t. Now, take the metric in constant $D + 1$ spacetime dimensions in the following form

$$ds^2 = -N^2(t)dt^2 + a^2(t)d\Sigma_k^2, \tag{19}$$

where $N(t)$ denotes the lapse function and $d\Sigma_k^2$ is the line element for a

D-manifold of constant curvature $k = +1, 0, -1$. The Ricci scalar is given by

$$R = \frac{D}{N^2} \left\{ \frac{2\ddot{a}}{a} + (D-1) \left[\left(\frac{\dot{a}}{a} \right)^2 + \frac{N^2 k}{a^2} \right] - \frac{2\dot{a}\dot{N}}{aN} \right\}. \tag{20}$$

Substituting from Eq.(20) in the Einstein-Hilbert action for pure gravity,

$$S_G = \frac{1}{2\kappa} \int d^{(1+D)} x \sqrt{-g} R, \tag{21}$$

and using the Hawking-Ellis action of a perfect fluid for the model universe with variable space dimension the following Lagrangian has been obtained for decrumpling or TVSD model (see Ref.[10])

$$L_I := -\frac{V_{D_t}}{2\kappa N} \left(\frac{a}{a_0} \right)^{D_t} D_t (D_t - 1) \left[\left(\frac{\dot{a}}{a} \right)^2 - \frac{N^2 k}{a^2} \right] - \rho N V_{D_t} \left(\frac{a}{a_0} \right)^{D_t}, \tag{22}$$

where $\kappa = 8\pi M_P^{-2} = 8\pi G$, ρ the energy density, and V_{D_t} the volume of the space-like sections

$$V_{D_t} = \frac{2\pi^{(D_t+1)/2}}{\Gamma[(D_t + 1)/2]}, \quad \text{closed Universe, } k = +1, \tag{23}$$

$$V_{D_t} = \frac{\pi^{(D_t/2)}}{\Gamma(D_t/2 + 1)} \chi_c^{D_t}, \quad \text{flat Universe, } k = 0, \tag{24}$$

$$V_{D_t} = \frac{2\pi^{(D_t/2)}}{\Gamma(D_t/2)} f(\chi_c), \quad \text{open Universe, } k = -1. \tag{25}$$

Here χ_C is a cut-off and $f(\chi_c)$ is a function thereof (see Ref. [10]).

In the limit of constant space dimensions, or $D_t = D_0$, L_I approaches to the Einstein-Hilbert Lagrangian which is

$$L_I^0 := -\frac{V_{D_0}}{2\kappa_0 N} \left(\frac{a}{a_0} \right)^{D_0} D_0 (D_0 - 1) \left[\left(\frac{\dot{a}}{a} \right)^2 - \frac{N^2 k}{a^2} \right] - \rho N V_{D_0} \left(\frac{a}{a_0} \right)^{D_0}, \tag{26}$$

where $\kappa_0 = 8\pi G_0$ and the zero subscript in G_0 denotes its present value. So, Lagrangian L_I cannot abandon Einstein's gravity. Varying the Lagrangian

L_I with respect to N and a, we find the following equations of motion in the gauge $N = 1$, respectively (see Ref.[10])

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{2\kappa\rho}{D_t(D_t - 1)}, \tag{27}$$

$$(D_t - 1)\left\{\frac{\ddot{a}}{a} + \left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right]\left(-\frac{D_t^2}{2C}\frac{d\ln V_{D_t}}{dD_t} - 1 - \frac{D_t(2D_t - 1)}{2C(D_t - 1)} + \frac{D_t^2}{2D_0}\right)\right\}$$
$$+ \kappa p\left(-\frac{d\ln V_{D_t}}{dD_t}\frac{D_t}{C} - \frac{D_t}{C}\ln\frac{a}{a_0} + 1\right) = 0. \tag{28}$$

Using (16) and (27), the evolution equation of the space dimension can be obtained by

$$\dot{D}_t^2 = \frac{D_t^4}{C^2}\left[\frac{2\kappa\rho}{D_t(D_t - 1)} - k\delta^{-2}e^{-2C/D_t}\right]. \tag{29}$$

The continuity equation of decrumpling or TVSD model can be obtained by (27) and (28)

$$\frac{d}{dt}\left[\rho\left(\frac{a}{a_0}\right)^{D_t}V_{D_t}\right] + p\frac{d}{dt}\left[\left(\frac{a}{a_0}\right)^{D_t}V_{D_t}\right] = 0. \tag{30}$$

In TVSD model, we can rewrite Eq.(10) in this form

$$G_{(3+1)} = \frac{S^{[D_t]}}{4\pi}\frac{G_{(D_t+1)}}{V^{[D_t-3]}}, \tag{31}$$

where $V^{[D_t-3]}$ is the volume of $(D_t - 3)$ extra spatial dimensions. One general feature of extra-dimensional theories, such as Kaluza-Klein and string theories, is that the "true" constants of nature are defined in the full higher dimensional theory so that the effective 4-dimensional constants depends, among other things, on the structure and size of the extra-dimensions. Any evolution of these sizes either in time or space, would lead to a spacetime dependence of the effective 4-dimensional constants, see Ref. [12]. So $G_{(D_t+1)}$ is a "true" constant and has variable dimension $[\text{length}]^{D_t-1}$. Therefore, we have $\dot{G}_{(D_t+1)} = 0$.

Using Eq.(31), the time variation of the gravitational Newton's constant in $(3 + 1)$-dimensional spacetime and in TVSD model is given by

$$\frac{\dot{G}_{(3+1)}}{G_{(3+1)}} = \frac{\dot{S}^{[D_t]}}{S^{[D_t]}} - \frac{\dot{V}^{[D_t-3]}}{V^{[D_t-3]}}. \tag{32}$$

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On the other hand, in TVSD model the surface area of a unit sphere in D_t-dimensional spaces is given by (see Eq.(5))

$$S^{[D_t]} = \frac{2\pi^{D_t/2}}{\Gamma\left(\frac{D_t}{2}\right)}. \tag{33}$$

Time derivative of this equation leads to

$$\frac{\dot{S}^{[D_t]}}{S^{[D_t]}} = \frac{\dot{D}_t}{2}\left[\ln\pi - \psi\left(\frac{D_t}{2}\right)\right], \tag{34}$$

where Euler's psi function ψ is the logarithmic derivative of the gamma function $\psi(x) \equiv \Gamma'(x)/\Gamma(x)$.

In TVSD model, the volume of $(D_t - 3)$ extra spatial dimensions is given by[4] (see Eq.(22))

$$V^{[D_t-3]} = \left(\frac{a}{a_0}\right)^{D_t-3}, \tag{35}$$

where zero subscript in a_0 denotes the present value of the scale factor. Therefore, we have

$$\frac{\dot{V}^{[D_t-3]}}{V^{[D_t-3]}} = \dot{D}_t \ln\left(\frac{a}{a_0}\right) + (D_t - 3)\frac{\dot{a}}{a}. \tag{36}$$

Using Eqs.(32), (34) and (36) we obtain

$$\frac{\dot{G}_{(3+1)}}{G_{(3+1)}} = \frac{\dot{D}_t}{2}\left[\ln\pi - \psi\left(\frac{D_t}{2}\right)\right] - \dot{D}_t \ln\left(\frac{a}{a_0}\right) - (D_t - 3)\frac{\dot{a}}{a}. \tag{37}$$

Let us obtain the time variation of the gravitational coupling constant in decrumpling or TVSD model. From Eq.(11), it can be easily obtained the gravitational coulping constant in TVSD model

$$\kappa_{(D_t+1)} = 4\pi(D_t - 1)G_{(3+1)}V^{[D_t-3]}. \tag{38}$$

Time derivative of Eq.(38) leads to

$$\frac{\dot{\kappa}_{(D_t+1)}}{\kappa_{(D_t+1)}} = \frac{\dot{D}_t}{(D_t - 1)} + \frac{\dot{G}_{(3+1)}}{G_{(3+1)}} + \frac{\dot{V}^{[D_t-3]}}{V^{[D_t-3]}}. \tag{39}$$

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[4]In Ref.[4], we considered $V^{[D_t-3]} \simeq a^{D_t-3}$. This makes some problems in calculations of Ref.[4]. For example, Eq.(37) in Ref.[4] must be corrected to Eq.(37) in this paper.

From Eqs.(32) and (39), we obtain

$$\frac{\dot{\kappa}_{(D_t+1)}}{\kappa_{(D_t+1)}} = \frac{\dot{D}_t}{(D_t - 1)} + \frac{\dot{S}^{[D_t]}}{S^{[D_t]}}. \tag{40}$$

Using Eqs.(34) and (40) we obtain

$$\frac{\dot{\kappa}_{(D_t+1)}}{\kappa_{(D_t+1)}} = \dot{D}_t \left[\frac{1}{D_t - 1} + \frac{\ln \pi}{2} - \frac{1}{2}\psi \left(\frac{D_t}{2} \right) \right]. \tag{41}$$

Let us now use Eqs.(37) and (41) at the present time. Taking $D_t|_0 = 3$, $a = a_0$ and using

$$\psi \left(\frac{3}{2} \right) = 2 - \gamma - 2\ln(2), \tag{42}$$

where γ is Euler's constant and approximately 0.5772156649, we obtain from Eqs.(37) and (41) respectively

$$\frac{\dot{G}_{(3+1)}}{G_{(3+1)}} \simeq 0.554 \, \dot{D}_t, \tag{43}$$

$$\frac{\dot{\kappa}_{(D_t+1)}}{\kappa_{(D_t+1)}} \simeq 1.054 \, \dot{D}_t. \tag{44}$$

According to Ref.[12], the absolute value of the time variation of the Newton's constant in three-space dimension today has an upper limit

$$\left| \frac{\dot{G}_{(3+1)}}{G_{(3+1)}} \right|_0 < 9 \times 10^{-12}\text{yr}^{-1}. \tag{45}$$

Using Eqs.(43) and (44) one gets

$$\left| \dot{D}_t \right|_0 < 1.6 \times 10^{-11}\text{yr}^{-1}. \tag{46}$$

Now, from Eqs.(44) and (46) we obtain

$$\left| \frac{\dot{\kappa}_{(D_t+1)}}{\kappa_{(D_t+1)}} \right|_0 < 1.7 \times 10^{-11}\text{yr}^{-1}. \tag{47}$$

4 Conclusions

In this paper, we study the time variation of the gravitational coupling constant in TVSD or decrumpling model. Using observational bounds on the time variation of the Newton's constant in three-space dimension we obtain a constraint on the absolute value of the time variation of the spatial dimension, see Eq.(46), and then a constraint on the absolute value of the time variation of the gravitational coupling constant in the model. It is worth mentioning that in Ref.[11], by using the observational bounds on the time variation of the fine structure constant, we have obtained a constraint on the absolute value of the time variation of the spatial dimension which is $|\dot{D}_t||_0 < 10^{-15} \mathrm{yr}^{-1}$. Comparing our result in this paper, i.e. Eq.(46), with the result presented in Ref.[11], one can conclude the absolute value of the time variation of the spatial dimension must be less than $10^{-15} \mathrm{yr}^{-1}$.

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